

# Home Assignment 2: Pupil Reflex Dynamics

October 6, 2017

*Preparation: exercises in Chapters 3-5 of the exercise manual.*

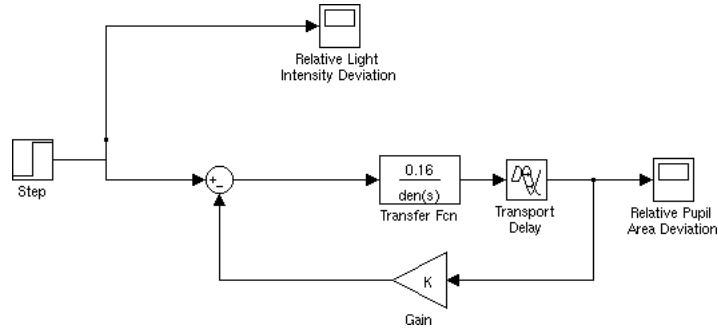
In the visual system, the light intensity adaption mechanism is partly controlled by adjusting the size of the iris. In this home assignment, we will investigate this phenomena by the model developed by Stark et al, based on open-loop experiments described in [3], and given in (1). The proposed model is a simplified linear model that serves as a decent approximation for signals with low magnitude. See [1, 2, 4] for more background on the model. The transfer function in (1) describes the relation between the relative increase in light intensity (input) and relative decrease in pupil area (output):

$$G(s) = \frac{Y(s)}{U(s)} = \frac{0.16e^{-0.18s}}{(1 + 0.1s)^3} \quad (1)$$

1. Give a state-space representation of the system. Confirm your result by computing the transfer function of your state-space representation using the `ss` and `tf` commands in MATLAB.
2. Plot the impulse response (`impz`), step response (`step`) and Bode plot (`bode`) of the open-loop system in (1). Can you give a physiological interpretation of each plot?
3. What is the amplitude margin of the system? Derive an analytic equation for the crossover frequency, solve the equation numerically and compute the value of the amplitude margin.

Normally the system (1) is in closed-loop, i.e., the closing or opening of the iris affects the amount of incoming light to the retina in a one-to-one ratio. Stark investigated the behavior of the pupil reflex by artificially increasing the gain of the feedback loop. He did this by shining a light beam at the edge of the iris, thereby altering the ratio between iris opening and the corresponding increase in light intensity on the retina.

4. Build a Simulink model of the closed-loop system as depicted in Fig. 1. In the top menu: go to **Simulation -> Configuration parameters** and set the step size fixed to 0.01 and the solver to ode4. Simulate the system with different values of the gain  $K$ . For instance, start with  $K = 0$  and see if you can reproduce the step response you plotted in step 2. Then, for increasing  $K$ , look at the output and determine at which gain the system becomes unstable.
5. How does the gain at which the system becomes unstable correspond to the Bode plot of the open-loop system? Compare your simulation results with the calculated amplitude margin.



**Figure 1** Closed-Loop System Model in Simulink

## 1. References

- [1] L. Stark and P. M. Sherman, "A servoanalytic study of consensual pupil reflex to light," *J. Neurophysiol*, vol. 20, pp. 17–26, 1957.
- [2] L. Stark, "Stability, oscillations, and noise in the human pupil servomechanism," *Proceedings of the IRE*, vol. 47, no. 11, pp. 1925–1939, 1959.
- [3] L. Stark and F. Baker, "Stability and oscillations in a neurological servomechanism," *Journal of neurophysiology*, vol. 22, no. 2, pp. 156–164, 1959.
- [4] L. W. Stark, "The pupil as a paradigm for neurological control systems," *Biomedical Engineering, IEEE Transactions on*, no. 12, pp. 919–924, 1984.