The Feedback Amplifier

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The Feedback Amplifier

K. J. Åström

Theme: Pure feedback.

Lectures

		1940	1960	2000
1	Introduction			
2	Governors	|	|	|
3	Process Control	|	|	|
4	Feedback Amplifiers	|	|	|
5	Harry Nyquist	|	|	|
6	Aerospace	|	|	|
		|		
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8	The Second Phase	←	←	|
9	The Third Phase	←	←	←
10	The Swedish Scene			
11	The Lund Scene			

The Power of Feedback

Feedback has some amazing properties, it can

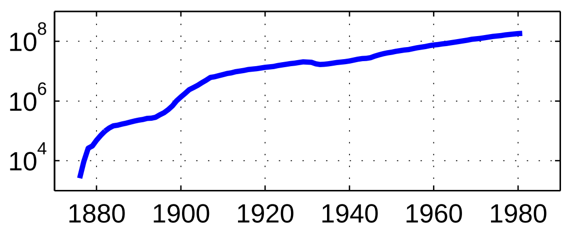
- make a system insensitive to disturbances,
- make good systems from bad components,
- follow command signals
- stabilize an unstable system,
- create desired behavior, for example linear behavior from nonlinear components.

The major drawbacks are that

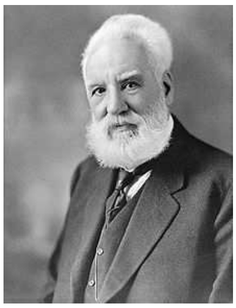
- feedback can cause instabilities
- sensor noise is fed into the system

Introduction

- Driving force: Telecommunications a rapidly growing industry
- Alexander Graham Bell 1847-1922
- Patent 1876
- No patent in Sweden!
- Lars Magnus Ericsson 1846-1926



Bell and Ericsson



The AT&T Research Laboratories

Bennet 2 p 70–71: *The AT&T Company formed an industrial research laboratory as part of its strategy of controlling all American telecommunications, summarized by its then President, Theodore Vail, as 'One Policy, One System, Universal Service'. To implement the strategy the company needed to control the rate and direction of technical change by obtaining, or preventing others from obtaining, key patents*; and it also needed to avoid being broken up under the Sherman Antitrust Act. *The research laboratories played a major part in ensuring that the company kept control of the technology and the patent rights.*

The transistor was also invented at Bell Labs!

The Repeater Problem

- The electro mechanical repeater
- 6mm wire 280 kg/km
- 1911 East coast to Denver
- 1914 First transcontinental New York San Francisco
- 1915 Improved transcontinental three vacuum tube repeaters, two repeaters added in 1916 and two more in 1918.

System	*Date*	*Ch pair*	*Loss db 3000mi*	*Repeat 3000mi*
1st TC	1914	1	60	3-6
2nd TC	1923	1-4	150-400	6-20
Open W	1938	16	1000	40
Cable	1936	12	12.000	200
Coaxial	1941	480	30.000	600

Bode - Feedback The History of an Idea

Bode, H. W. Feedback - The History of an idea. Proc. Symp Active Networks and Feedback Systems. Polytechnic Institute of Brooklyn, 1960. in Bellman and Kalaba. Selected papers on mathematical trends in control theory. Dover 1964.

Most of you with hi-fi systems are no doubt proud of your audio amplifiers, but I doubt whether many of you would care to listen to the sound after the signal had gone in succession through several dozen or several hundred even of your fine amplifiers. There is a 'tyranny of numbers, as my reliability friends say, which makes it necessary for the individual components of the system to become qualitatively better as the system as a whole becomes quantitatively more ambitious.

Bode - Feedback The History of an Idea

The causes of distortion were of various sorts. They included power supply noises, variations in gain and so on. The dominant problem, however, was the inter-modulation due to the slight nonlinearity in the characteristics of the last tube. Various efforts were made to improve this situation, by the selection of tubes, by careful biasing, by the use of matched tubes in push-pull to provide compensating characteristics, and so on. Until Black's invention, however, nothing made a radical improvement of the situation.

Distortion in Cascaded Amplifiers

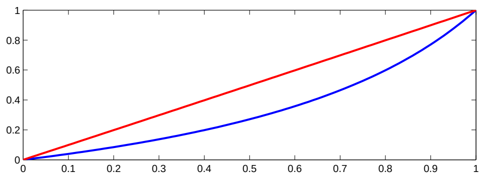
Compositions of functions

$$f_n = f \circ f \ldots \circ f$$

Define distortion as

$$d_n = 2\frac{\max f_n'(x) - \min f_n'(x)}{\max f_n'(x) + \min f_n'(x)}$$

Example $f(x) = \dfrac{x + ax^2}{1+a}$, $a = 0.01, \Rightarrow d_1 = 0.01, d_{100} = 0.74$



Audio Demo

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6. Summary

Theme: Pure feedback.

Harold S. Black

- Worcester Polytechnic Institute 1921
- Joined Western Electric that later became part of Bell Telephone Laboratories
- Remained with Bell until 1963
- Principal Research Scientist with General Precision Corporation

- The feedback amplifier
- Theory and application of pulse-code modulation
- IEEE Lamme Medal 1957
- 62 US Patents
- 271 patents in 32 countries
- 1953 Modulation Theory

Black on Black 1

H. S. Black, Inventing the negative feedback amplifier. IEEE Spectrum **14**(1977) 55–60.

The telephone industry was an exciting place for a young engineer in 1921. Only a few years had passed since Lee de Forest's 1906 audion tube had been made into the practical high-vacuum device needed for long-distance telephone lines. In 1915, the 68-year-old Alexander Graham Bell had placed the first official transcontinental call to Thomas Watson, his famous assistant 40 years earlier, and in 1914, a young Edwin Howard Armstrong had sat through bitterly cold January night with David Sarnoff in an American Marconi wireless shack testing Armstrong's new "regenerative" receiver, which utilized positive feedback.

Lesson learned: Look for exciting emerging technology

Black on Black 2

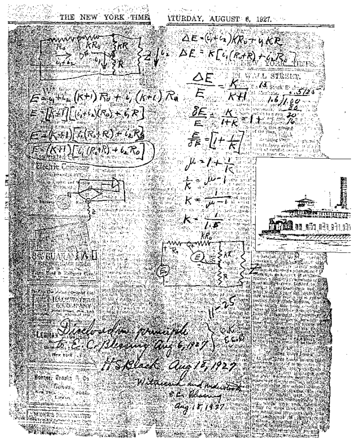
Few rosier dreams could be dreamed than that of an amplifier whose overall performance is perfectly constant, and in whose output distortion constitutes only one-hundred millionth of total energy, although the component parts may be far from linear in their response and their gain may vary over a considerable range. But the dreamer who awakes in amazement to find that such an amplifier can be built has additional surprises in store for him. These benefits can be obtained by simply throwing away some gain, and by utilizing feedback action.

Black on Black 3

Then came the morning of Tuesday, August 2, 1927, when the concept of the negative feedback amplifier came to me in a flash while I was crossing the Hudson River on the Lackawanna Ferry, on my way to work. For more than 50 years I have pondered how and why the idea came, and I can't say any more today than I could that morning. All I know is that after several years of hard work on the problem, I suddenly realized that if I fed the amplifier output back to the input, in reverse phase, and kept the device from oscillating (singing, as we called it then), I would have exactly what I wanted: a means of canceling out the distortion in the output. I opened my morning newspaper and on a page of The New York Times I sketched a simple canonical diagram ... when I reached the laboratory at 463 West Street, it was witnessed, understood, and signed by the late Earl C. Blessing.

Black's Original

- Patents
- Disclosures
- Engineering notebooks



The Idea of Feedback

However, by building an amplifier whose gain is deliberately made, say 40 decibels higher than necessary and then feeding the output back on the input in such a way as to throw away the excess gain, it had been found possible to effect extraordinary improvement in constancy of amplification and freedom from non-linearity.

Stabilized feedback processes other advantages including reduced delay and delay distortion, reduced noise disturbance from the power supply circuits and various other features best appreciated by practical designers of amplifiers.

$$\frac{Y}{U} = \frac{\mu}{1-\beta\mu} = -\frac{1}{\beta}\,\frac{\mu}{\mu - 1/\beta}$$

Gain is the hard-currency that can be traded for many other qualities!

Nine years in the Patent Office

Although the invention had been submitted to the U.S. Patent Office on August 8, 1928, more than nine years would elapse before the patent was issued on December 21, 1937 (No. 2 102 671). One reason for the delay was that the concept was so contrary to established beliefs that the Patent Office initially did not believe it would work. The Office cited technical papers, for example, that maintained the output could not be connected back to the input unless the loop gain was less than one, whereas mine was between 40 and 50 dB. In England, our patent application was treated in the same manner as one for a perpetual-motion machine. Burgess was eventually able to overcome all these objections by submitting evidence that 70 amplifiers were working successfully ...

Mervin Kelly on Black IEEE Lamme Medal 1957

Although may of Harold's inventions have made great impact, that of the negative feedback amplifier is indeed the most outstanding. It easily ranks coordinate with De Forest's invention of the audion as one of the two inventions of broadest scope and significance in electronics and communications of the past 50 years....it is no exaggeration to say that without Black's invention, the present long-distance telephone and television networks which cover our entire country and the transoceanic telephone cables would not exist. The application of Black's principle of negative feedback has not been limited to telecommunications. Many of the industrial and military amplifiers would not be possible except for its use.

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Hendrik Bode 1905-1982

- Born Madison Wisconsin
- Child protégée, father prof at UIUC, finished high school at 14
- Too young to enter UIUC
- Ohio State BA 1924, MA 1926 (Math)
- Bell Labs 1929
  - Network theory
  - Missile systems
  - Information theory
- PhD Physics Columbia 1936
- Gordon McKay Prof of Systems Engineering at Harvard 1967 (Bryson and Brockett held this chair later)

Major Contributions

Original sources

- H. W. Bode Network Analysis and Feedback Amplifier Design. Van Nostrand, Princeton 1945
- Bode, H. W. Feedback - The History of an idea. Proc. Symp Active Networks and Feedback Systems. Polytechnic Institute of Brooklyn, 1960. in Bellman and Kalaba. Selected papers on mathematical trends in control theory. Dover 1964.

Lasting contributions

- The Bode plot
- Gain-phase relations
- The notion of minimum phase and fundamental limitations
- The phase area formula
- The ideal loop transfer function (cut-off characteristic)

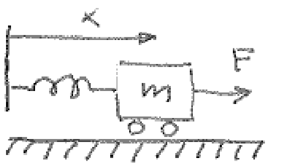
Bode on ME and EE

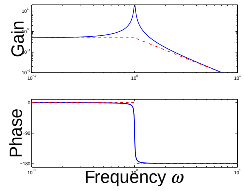
The two fields are radically different in character and emphasis. ... The fields also differ radically in their mathematical flavor. The typical regulator system can frequently be described, in essentials, by differential equations by no more than perhaps the second, third or fourth order. On the other hand, the system is usually highly nonlinear, so that even at this level of complexity the difficulties of analysis may be very great. ... As a matter of idle, curiosity, I once counted to find out what the order of the set of equations in an amplifier I had just designed would have been, if I had worked with the differential equations directly. It turned out to be 55

Åström Murray pp 28–32

The Bode Plot

- Logarithmic scales gives an overview of the behavior over wide frequency and amplitude ranges
- Piece-wise linear approximations admits good interpretations.





- Low frequencies $G_{xF}(s) \approx 1/k$, the spring line, system behaves like a spring for low frequency excitation.
- High frequencies $G_{xF}(s) \approx 1/(ms^2)$, the mass line,, system behaves like a mass for high frequency excitation.

Bode's Relations between Gain and Phase

While no unique relation between attenuation and phase can be stated for a general circuit, a unique relation does exist between any given loss characteristic and the *minimum phase shift* which must be associated with it. (p. 424)

$$\arg G(i\omega_0) = \frac{2\omega_0}{\pi}\int_0^\infty \frac{\log|G(i\omega)| - \log|G(i\omega_0)|}{\omega^2-\omega_0^2}d\omega$$
$$= \frac{1}{\pi}\int_0^\infty \frac{d\log|G(i\omega)|}{d\log\omega}\log\left|\frac{\omega+\omega_0}{\omega-\omega_0}\right|d\omega \approx \frac{\pi}{2}\frac{d\log|G(i\omega)|}{d\log\omega}$$

$$\frac{\log|G(i\omega)|}{\log|G(i\omega_0)|} = -\frac{2\omega_0^2}{\pi}\int_0^\infty \frac{\omega^{-1}\arg G(i\omega) - \omega_0^{-1}\arg G(i\omega_0)}{\omega^2-\omega_0^2}d\omega$$
$$= -\frac{2\omega_0^2}{\pi}\int_0^\infty \frac{d(\omega^{-1}\arg G(i\omega))}{d\omega}\log\left|\frac{\omega+\omega_0}{\omega-\omega_0}\right|d\omega$$

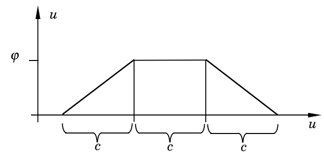
Bode's Phase Area Formula

Let $G(s)$ be a transfer function with no poles and zeros in the right half plane. Assume that $\lim_{s\to\infty} G(s) = G_\infty$. Then

$$\ln\frac{G_\infty}{G(0)} = \frac{2}{\pi}\int_0^\infty \arg G(i\omega)\frac{d\omega}{\omega} = \frac{2}{\pi}\int_{-\infty}^\infty \arg G(i\omega) d\log\omega$$

The gain K required to obtain a given phase lead φ is an exponential function of the area under the phase curve in the Bode plot

$$K = e^{4c\varphi_0/\pi} = e^{2\gamma\varphi_0}$$
$$\gamma = \frac{2c}{\pi}$$

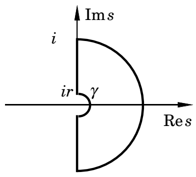


Proof

Integrate the function

$$\frac{\log G(s)/G(\infty)}{s}$$

around the contour, $\arg G(i\omega)/\omega$ even fcn

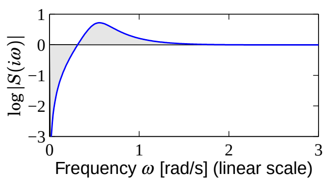


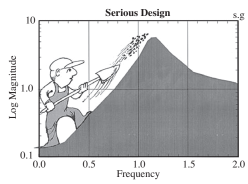
$$0 = \int_{-\infty}^0 \left(\log\frac{|G(\omega)|}{|G(\infty)|} + i\arg\frac{G(\omega)}{G(\infty)}\right)\frac{d\omega}{\omega} +$$
$$\int_0^{-\infty}\left(\log\frac{|G(\omega)|}{|G(\infty)|} + i\arg\frac{G(\omega)}{G(\infty)}\right)\frac{d\omega}{\omega} + i\pi\log\frac{|G(0)|}{|G(\infty)|}$$

Hence

$$\log\frac{|G(0)|}{|G(\infty)|} = \frac{2}{\pi}\int_0^\infty \arg G(i\omega)\, d\log\omega$$

Bode's Integral - The Waterbed Effect





$$\int_0^\infty \log|S(i\omega)|d\omega = \pi\sum \mathrm{Re}\, p_k - \frac{\pi}{2}\lim_{s\to\infty} sL(s)$$

The sensitivity can be decreased at one frequency at the cost of increasing it at another frequency.

Feedback design is a trade-off

Bode's Ideal Cut-off Characteristics 1

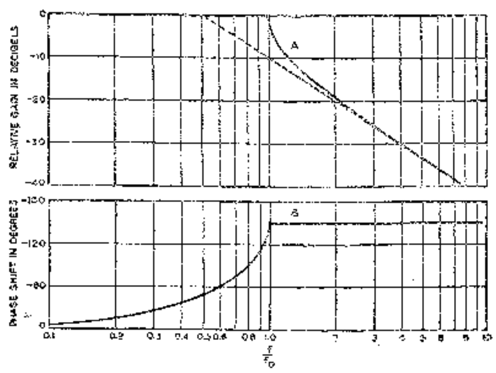
The essential feature is that the gain around the feedback loop be reduced from the large value which it has in the useful frequency band to zero or less at some higher frequency without producing an accompanying phase shift larger than some prescribed amount. ...

If it were not for the phase restriction it would be desirable on engineering grounds to reduce the gain very rapidly. The more rapidly the feedback vanishes for example, the narrower we need make the region in which active design attention is required to prevent singing. ...

But the analysis in Chapter XIV (Bode's relations) shows that the phase shift is broadly proportional to the rate at which the gain changes. ...

A phase margin of 30° correspond to a slope of -5/3.

Bode's Ideal Cut-off Characteristics 2



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Theme: Pure feedback.

Harry Nyquist 1889-1976

From farm life in Nilsby Värmland to Bell Labs

Dreaming to be a teacher

- Emigrated 1907
- High school teacher 1912
- MS EE U North Dakota 1914
- PhD Physics Yale 1917
- Bell Labs 1917

Key contributions

- Johnson-Nyquist noise
- The Nyquist frequency
- Nyquist's stability theorem

The Motivation 1

Mr. Black proposed a negative feedback repeater and proved by tests that it possessed the advantages predicted for it. In particular, its gain was constant to a high degree, and it was linear enough ...

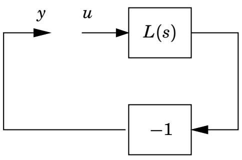
For best results, the feedback gain factor, the quantity usually known as $\mu\beta$ (the loop transfer function $L(s)$) had to be numerically much larger than unity. The possibility of stability with a feedback factor greater than unity was puzzling. Granted that the factor is negative it was not obvious how it would help. If the factor was -10 the effect of one round trip around the feedback loop is to change the original current from, say 1 to -10. After a second trip around the loop the current becomes 100, and so forth. The totality looks much like a diverging series and it was not clear how such a succession of ever-increasing components could add to something finite and so stable as experience had shown. ...

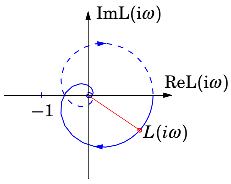
The Motivation 2

The missing part in this argument is that the numbers that describe the successive components 1, -10, 100, and so on, represents the steady state, whereas at any finite time many of the components have not yet reached steady state and some of them, which are destined to become very large, have barely reached perceptible magnitude. My calculations were principally concerned with replacing the infinite divergent series referred to by a series which give the actual value attained at a specific time t. The series thus obtained is convergent instead of divergent and, moreover converges to values in agreement with experimental findings.

A much simpler proof was later given based on Rouche's theorem

Condition for Oscillations





Cut the loop. Let u be a sinusoid. If y is a sinusoid with the same amplitude and phase, then the loop can be closed and the oscillation will be maintained. The condition for this is

$$L(i\omega) = -1$$

where $L(s) = P(s)C(s)$ is the loop transfer function. The condition implies that the Nyquist curve of $L(s)$ goes through the point -1 (the critical point)!

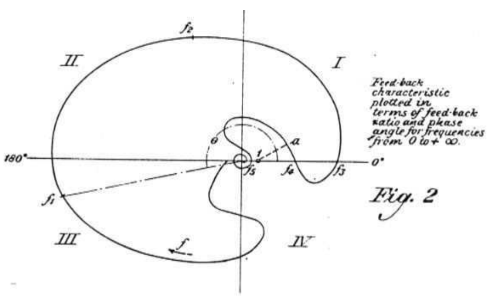
Nyquist Regeneration Theory BSTJ 1932

A Paradigm Shift: The input-output view instead of the state view and the characteristic equation

Stability Concept The circuit will be said to be stable when an impressed small disturbance, which itself dies out, results in a response which dies out. It will be said to be unstable when such a disturbance results in a response which goes on indefinitely, either staying at a relatively small value or increasing.

Rule: Plot plus and minus the imaginary part of $AJ(i\omega)$ against the real part for all frequencies from 0 to ω. If the point $1 + i0$ lies completely outside this curve the system is stable; if not it is unstable (p. 136).

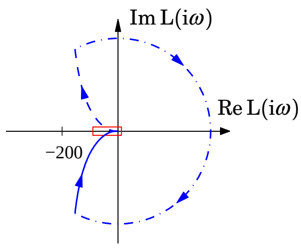
The Original Nyquist Curve

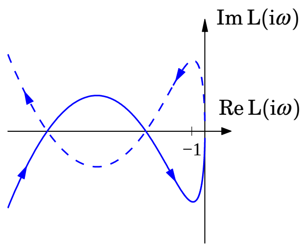


Bode moved the critical point to -1

Conditional Stability

It should perhaps be explained also how it comes to be so detailed. In the course of the calculations, the facts with which the term conditional stability have come to be associated, becomes apparent. One aspect of this is that it is possible to have a feedback loop which is stable and can be made unstable by increasing the loop gain. This seemed a very surprising result and appeared to require that all the steps be examined and set forth in full detail.





Impact of the Nyquist Theorem at ASEA

We had designed controllers by making simplified models, applying intuition and analyzing stability by solving the characteristic equation. (At that time, around 1950, solving the characteristic equation with a mechanical calculator was itself an ordeal.) If the system was unstable we were at a loss, we did not know how to modify the controller to make the system stable. The Nyquist theorem was a revolution for us. By drawing the Nyquist curve we got a very effective way to design the system because we know the frequency range which was critical and we got a good feel for how the controller should be modified to make the system stable. We could either add a compensator or we could use extra sensor.

Free translation from seminar by Erik Persson ABB in Lund 1970.
Why did it take 18 years?

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More Recent Developments

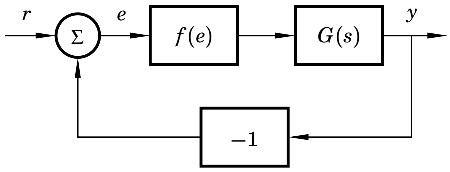
Brief summary

- Driving force: Rapid growth of telephone industry
- The key ideas were shaped by 1940, A comprehensive account in Bode's book
- The result had a major impact for the formation of automatic control which will be covered in lecture on servomechanisms

Some interesting later developments

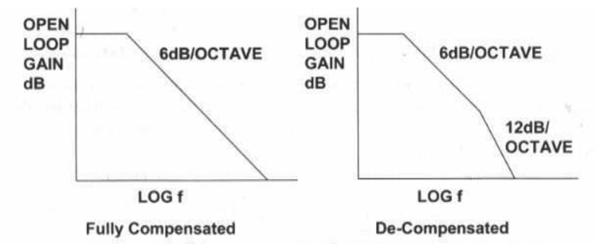
- Input output stability
  Popov, circle criterion, small gain theorem, passivity
- The operational amplifier
- Loop shaping
- Fractional systems and QFT
- Fundamental limitations

A Special Nonlinear System



- Lurje 1944
- Aizermans conjecture 1964
- Popov (absolute stability) 1961
- Yakubovich 1962 (LMI) Kalman 1963
- Zames 1964 Sandberg 1964
- Circle criterion, small gain theorem, passivity

The Operational Amplifier



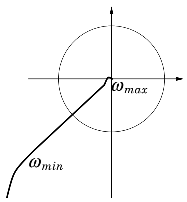
- Great effort is made to make the gain high ($> 10^7$) and the open loop transfer function close to $G(s) = 1/s$
- Easy to close feedback around operational amplifiers
- Core of analog computing

Loop Shaping for Gain Variations

The repeater problem. Large gain variations in vacuum tube amplifiers. The loop transfer function

$$L(s) = \left(\frac{s}{\omega_{gc}}\right)^n$$

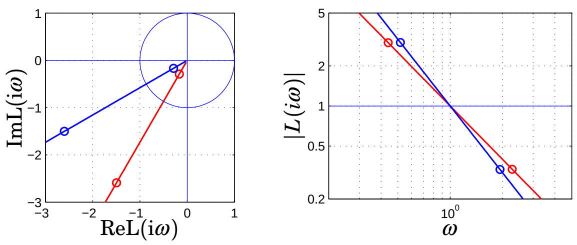
gives a phase margin that is invariant to gain variations.



The slope $n = -1.5$ gives the phase margin $\varphi_m = 45°$.

Horowitz extended Bodes ideas to deal with arbitrary plant variations not just gain variations in the QFT method.

Trade-offs



- Blue curve slope $n = -5/3$ phase margin $\varphi_m = 30°$
- Red curve slope $n = -4/3$ phase margin $\varphi_m = 60°$
- Making the curve steeper reduces the frequency range where compensation is required but the phase margin is reduced

Fractional Transfer Functions 1

Consider the process

$$P(s) = \frac{1}{s(s+1)}$$

Find a controller that gives $L(s) = s^{-1.5}$. Hence

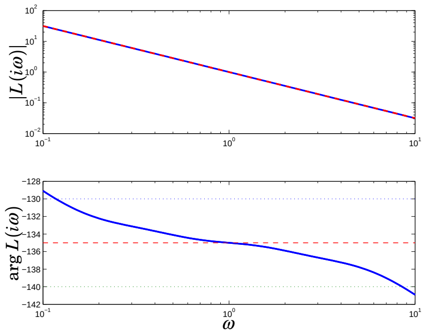
$$C(s) = \frac{L(s)}{P(s)} = \frac{s(s+1)}{s\sqrt{s}} = \sqrt{s} + \frac{1}{\sqrt{s}}$$

A controller with fractional transfer function. To implement it we approximate by a rational transfer function

$$\hat{C}(s) = k\frac{(s+1/16)(s+1/4)(s+1)(s+4)(s+16)}{(s+1/32)(s+1/8)(s+1/2)(s+2)(s+8)(s+32)}$$

High controller order gives robustness

A Fractional Transfer Functions 2



The phase margin changes only by 5° when the process gain varies in the range 0.03-30! Horowitz QFT is a generalization!

The Gain Crossover Inequality

Factor process transfer function as $P(s) = P_{mp}(s)P_{nmp}(s)$ such that $|P_{nmp}(i\omega)| = 1$ and P_{nmp} has negative phase. Requiring a phase margin φ_m we get

$$\begin{aligned}\arg L(i\omega_{gc}) &= \arg P_{nmp}(i\omega_{gc}) + \arg P_{mp}(i\omega_{gc}) + \arg C(i\omega_{gc}) \\ &\geq -\pi + \varphi_m\end{aligned}$$

Approximate $\arg\left(P_{mp}(i\omega_{gc})C(i\omega_{gc})\right) \approx n\pi/2$ gives

$$\arg P_{nmp}(i\omega_{gc}) \geq -\pi + \varphi_m - n\frac{\pi}{2}$$

This equation called, the gain crossover inequality. Equality holds exactly if $P_{nmp}C$ is Bode's ideal loop transfer function, the expression is an approximation for other designs if n is the slope at the crossover frequency.

Non-minimum Phase Limits Performance

The inequality

$$\arg P_{nmp}(i\omega_{gc}) \geq -\pi + \varphi_m - n_{gc}\frac{\pi}{2}$$

says that the phase lag of the non-minimum phase component cannot be too large at the crossover frequency!

Simple rules of thumb:

- $\varphi_m = 45°$ and $n_{gc} = -1$ gives

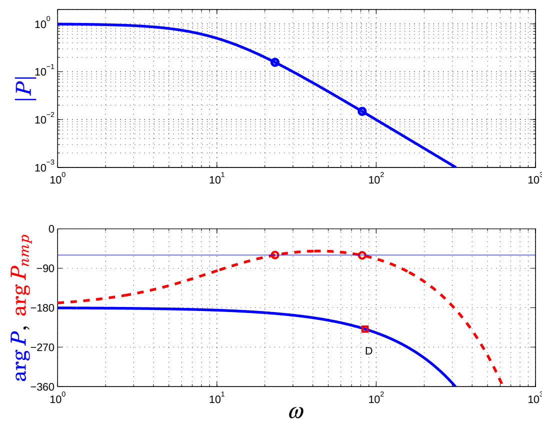
$$\arg P_{nmp}(i\omega_{gc}) \geq -\frac{\pi}{4} = -0.8\ [rad], \quad 45°$$

- $\varphi_m = 45°$ and $n_{gc} = -0.5$ gives

$$\arg P_{nmp}(i\omega_{gc}) \geq -\frac{\pi}{2} = -1.6\ [rad], \quad 90°$$

Non-minimum phase components can have a phase-lag of at most 45° – 90° at the gain cross over frequency!

Bode Plots Should Look Like This



The Feedback Amplifier

K. J. Åström

1. Introduction
2. Black's Invention
3. Bode
4. Nyquist
5. More Recent Developments
6. Summary

Theme: Pure feedback.

Summary

- An industrially driven development with large impact
- Driving force: the telephone industry
  - The monopoly and wise leadership
  - The AT&T Research Lab (Bell Labs)
- Differences between telecommunication, steam engines, hydroelectric power generation and process control
  - Bell Labs versus 600 instrument company
  - Amplifiers are simpler than steam engines
- Strong theoretical development
  - Understanding feedback
  - Stability theory
  - Input-output view frequency response
  - The power of theory of complex variables
  - Design methods
  - Fundamental limitations (non-minimum phase)
- Cornerstone in creation of the field of Control

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